Sec. 8.2 Sinusoidal Functions and Their Graphs

Theorem: For the graphs of $y = A\sin(Bx+C)+D$ or y = $A\cos(Bx+C)+D$,

a. Amplitude = $|A|$

b. Period = $P = \dfrac{2\pi}{|B|}$

c. Phase Shift = $-C$

d. Horizontal Shift = $\dfrac{-C}{|B|}$

e. Midline = horizontal line $y = D$

f. Frequency = number of cycles per unit time = $\dfrac{|B|}{2\pi} = \dfrac{1}{P}$

Some mathematics texts treat phase shift and horizontal shift as being the same thing. This one does not.

Horizontal Shift – the distance a graph's starting value (at $x = 0$) is shifted left or right.

Phase Shift – allows us to calculate the fraction of a full period that a curve has been shifted. Phase Shift = Fraction of Period $\times\, 2\pi$.

You can also use transformations to find the midline, amplitude, and horizontal shift. But keep in mind that horizontal shift is the result of a horizontal translation and a horizontal stretch/compression. So, if you want $-C$ to represent the horizontal shift, the stretch/compression must be factored out first. So, if:

$$y = A\sin(B(x+C))+D \qquad\qquad y = A\cos(B(x+C))+D$$

Then Horizontal Shift = $-C$ Phase Shift = $-BC$

Phase shift is easiest to find in non-factored form, while horizontal shift is easiest to find in factored form.

Ex. State the midline and amplitude of the following sinusoidal functions:

(a) $y = 3\sin t + 5$

midline: $y = 5$

amplitude = 3

(b) $y = \dfrac{4 - 3\cos t}{20}$

$y = \frac{4}{20} - \frac{3\cos t}{20}$

$y = \frac{1}{5} - \frac{3}{20}\cos t$

midline: $y = \frac{1}{5}$

amplitude: $\frac{3}{20}$

Ex: Find the amplitude, period, horizontal shift, and phase shift of the following functions:

(a) $y = -3\sin(4x+2)$ $\quad y=-3\sin\left(4\left(x+\frac{1}{2}\right)\right)$

Amplitude = 3

Period = $\frac{2\pi}{4} = \frac{\pi}{2}$

HORIZONTAL SHIFT = $\frac{-C}{B}$

$= \frac{-2}{4} = -\frac{1}{2}$

Phase Shift = −2

(b) $y = \frac{1}{2}\cos(\pi x - 2\pi)$ $\quad y = \frac{1}{2}\cos\left(\pi(x-2)\right)$

Amplitude = $\frac{1}{2}$

Period = $\frac{2\pi}{\pi} = 2$

HORIZONTAL SHIFT = 2

Phase Shift = 2π

Steps for Graphing Sinusoidal Functions:

1. Determine the amplitude, $|A|$ and period, $\frac{2\pi}{|B|}$.
2. Determine the starting point of one cycle of the graph, $\frac{-C}{B}$.
3. Determine the ending point of one cycle of the graph, $\frac{-C}{B} + \frac{2\pi}{|B|}$.
4. Divide the interval into four subintervals of equal length $\frac{2\pi}{|B|} \div 4$.
5. Use the endpoints of the subintervals to find the five key points on the graph.
6. Fill in one cycle of the graph.
7. Extend the graph in each direction to make it complete.

Ex. Find the amplitude, midline, period, horizontal shift, and phase shift of $y = 3\sin(2x - \pi)$ and then graph the function. $\quad y = 3\sin\left(2\left(x-\frac{\pi}{2}\right)\right)$

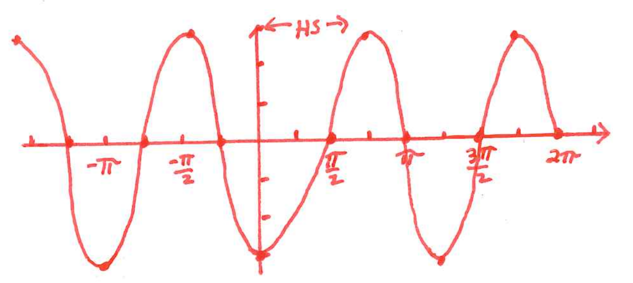
Amplitude = 3

Period = $\frac{2\pi}{2} = \pi$ $\quad \left(\pi \cdot \frac{1}{4} = \frac{\pi}{4}\right)$

Midline ⇒ $y = 0$

Horizontal Shift = $\frac{\pi}{2}$

Phase Shift = π



Phase Shift = Fraction of Per × 2π

π = F of P × 2π

$\frac{\pi}{2\pi}$ = F of P

$\frac{1}{2}$ = Fraction of Period

$\frac{1}{2} \cdot \pi = \frac{\pi}{2}$

(P) (HS)

Ex. Find the amplitude, midline, period, horizontal shift, phase shift of $y = 2\cos(4x + 3\pi) - 2$ and then graph the function. $\quad y = 2\cos\left(4\left(x+\frac{3\pi}{4}\right)\right)-2$

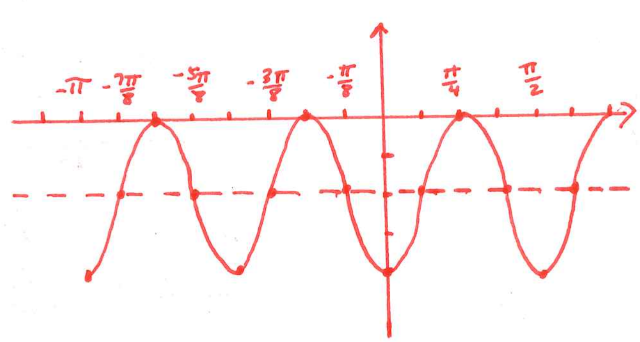
Amplitude = 2

Period = $\frac{2\pi}{4} = \frac{\pi}{2}$ $\quad \left(\frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}\right)$

Midline: $y = -2$

Horizontal Shift = $-\frac{3\pi}{4}$

Phase Shift = -3π



PS = F of P × 2π

-3π = F of P × 2π

$-\frac{3\pi}{2\pi}$ = F of P

$-\frac{3}{2}$ = F of P

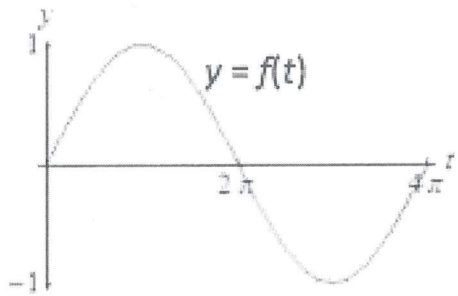
$-\frac{3}{2} \cdot \frac{\pi}{2} = -\frac{3\pi}{4}$

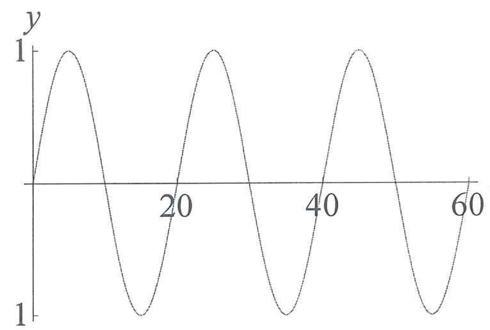
(P) (HS)

Steps for Fitting Data to a Sine Function y = $A\sin(Bx+C)+D$ **.**

1. Determine A, the amplitude of the function, by using
   Amplitude = (largest data value – smallest data value)/2
2. Determine *k*, the vertical shift of the function by using
   Vertical shift = (largest data value + smallest data value)/2
3. Determine *B*. Since the period *P*, the time it takes for the data to repeat is
   $P = \dfrac{2\pi}{|B|}$, we have $B = 2\pi/P$.
4. Determine the horizontal shift of the function by solving the equation
   $y = A\sin(Bx+C)+D$ for *h* by choosing an ordered pair (x,y) from the data. Since answers would vary depending on ordered pair chosen, always choose the pair for which y is smallest in order to maintain consistency.

Ex: Write the equation for the following sine or cosine waves.





Sine
Amplitude = 1
Period = 4π
Midline = $y = 0$
Reflection: NO
HS: NO

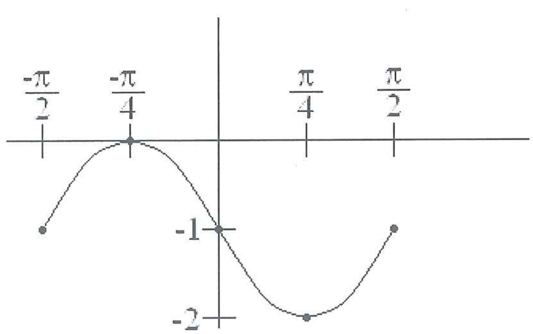
$P = \frac{2\pi}{B}$
$4\pi = \frac{2\pi}{B}$
$4\pi B = 2\pi$
$B = \frac{1}{2}$

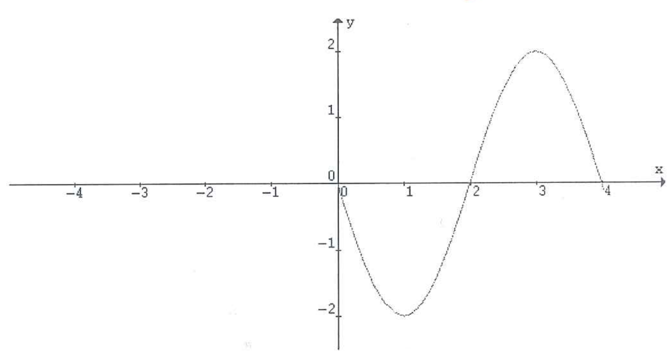
$y = \sin\left(\frac{1}{2}t\right)$

Sine
Amplitude = 1
Period = 20
Midline $y = 0$

Reflection: No
HS: No

$B = \frac{2\pi}{P}$
$B = \frac{2\pi}{20} = \frac{\pi}{10}$
$y = \sin\left(\frac{\pi}{10}x\right)$





Sine
Amplitude = 1
Period = π
Midline: $y = -1$
Reflection: NO
HS: $-\frac{\pi}{2}$

$B = \frac{2\pi}{\pi}$
$B = 2$

$y = \sin\left(2\left(x + \frac{\pi}{2}\right)\right) - 1$
or
$y = \sin(2x + \pi) - 1$

Sine
Amplitude = 2
Period = 4
Midline = $y = 0$
Reflection: YES
HS: NO

$B = \frac{2\pi}{4} = B = \frac{\pi}{2}$

$y = -2\sin\left(\frac{\pi}{2}x\right)$

HW: pg 330-333 # 2, 4-6, 10-12, 15, 16, 18, 22, 23, 33ab, 35, 36